

1 Additional Exercises: Products of Groups

1. Let G, G' be groups. What is the order of the product group $G \times G'$?
2. Is the symmetric group S_3 a direct product HK of non-trivial subgroups?
3. Prove a finite cyclic group of order rs is isomorphic to the product of cyclic groups of orders r and s if and only if r and s have no common factor.
 - (a) Let H, K be subgroups of a group G . Show the set of products $HK = \{hk : h \in H, k \in K\}$ is a subgroup if and only if $HK = KH$.
 - (b) Give an example of a group G and two subgroups H, K such that HK is not a subgroup.
4. Let $x \in G$ have order m and $y \in G'$ have order n . What is the order of $(x, y) \in G \times G'$?
5. Let G be a group containing normal subgroups of orders 3 and 5, respectively. Prove G contains an element of order 15.